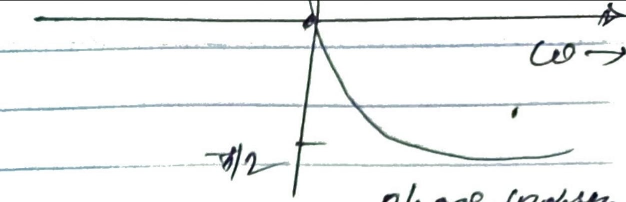
Fig: Amplitude spectrum



phase spectrum

Properties of Continuous time Fourier Transform.

(1) Linearity property:

The linearity properties states that the weighted sum of two signal is equal to weighted sum of their individual transform.

i.e If

$$x_1(t) \xleftrightarrow{FT} X_1(\omega)$$

and $$x_2(t) \longleftrightarrow X_2(\omega)$$

Then,

$$\boxed{a\,x_1(t) + b\,x_2(t) \xleftrightarrow{FT} a\,X_1(\omega) + b\,X_2(\omega).}$$

where a and b are constants

Proof:

$$F[a\,x_1(t) + b\,x_2(t)] = \int_{-\infty}^{+\infty} [a x_1(t) + b x_2(t)] \cdot e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{+\infty} a\, x_1(t) e^{-j\omega t}\,dt + b\int_{-\infty}^{\infty} x_2(t) \cdot e^{-j\omega t}\,dt$$

$$= a\,X_1(\omega) + b\,X_2(\omega).$$

Proved

(2) Time shifting property:

The time shifting property states that if a signal $x(t)$ is shifted by t_0 sec, the spectrum is modified by a linear phase shift of slope $-\omega t_0$

i.e If $x(t) \xleftrightarrow{FT} X(\omega)$

Then, $\boxed{x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega).}$

proof: $\because$ $F[x(t)] = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt$

now, delay the signal by t_0 times, $x(t) \xrightarrow{\text{delay } t_0} x(t-t_0)$.

$$F[x(t-t_0)] = \int_{-\infty}^{+\infty} x(t-t_0) \cdot e^{-j\omega t} dt$$

put $t - t_0 = p$

$t = t_0 + p$

$$F[x(p)] = \int_{-\infty}^{+\infty} x(p) \cdot e^{-j\omega(t_0+p)}$$

$$= \int_{-\infty}^{+\infty} x(p) \cdot e^{-j\omega t_0} \cdot e^{-j\omega p} \cdot dp = \int_{-\infty}^{+\infty} x(p) e^{-j\omega p} dp$$

$$F[x(t-t_0)] = X(\omega) \cdot e^{-j\omega t_0}$$

$\therefore$ $x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega)$.

Similarly $\boxed{x(t+t_0) \xleftrightarrow{FT} e^{j\omega t_0} \cdot X(\omega).}$ Proved

Here There is no change in magnitude spectrum but the phase spectrum is linearly shifted.

(3) Frequency shifting property (Multiplication by an exponential)

Frequency shifting property states that the multiplication of a time domain signal $x(t)$ by $e^{j\omega_0 t}$ results in frequency spectrum shifted by ω_0 i.e

If $x(t) \xleftrightarrow{FT} X(\omega)$.

Then, $\boxed{e^{j\omega_0 t} x(t) \longleftrightarrow X(\omega-\omega_0).}$

proof:

$\because \quad F[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

then, $F[e^{j\omega_0 t} x(t)] = \int_{-\infty}^{+\infty} e^{j\omega_0 t} x(t) e^{-j\omega t} dt$

$= \int_{-\infty}^{+\infty} e^{j(\omega_0 - \omega)t} dt$

$= \int_{-\infty}^{+\infty} e^{-j(\omega - \omega_0)t} dt$

$= X(\omega - \omega_0)$

$\therefore \quad \boxed{e^{j\omega_0 t} x(t) \xleftrightarrow{FT} X(\omega - \omega_0).}$

$e^{-j\omega t_0}]$ Similarly $\boxed{e^{-j\omega_0 t} x(t) \longleftrightarrow X(\omega + \omega_0)}$ Proved

(4) Time Reversal property:

· The time reversal property states that

If $x(t) \xleftrightarrow{FT} X(\omega)$.

Then $\boxed{x(-t) \xleftrightarrow{FT} X(-\omega).}$

proof:

$\because \quad F[x(t)] = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt$

then

$F[x(-t)] = \int_{-\infty}^{+\infty} x(-t) \cdot e^{-j\omega t} dt$

$= X(-\omega)$. Proved.

(5) Time scaling property:

If $x(t) \xleftrightarrow{FT} X(\omega)$.

then $\boxed{x(at) \xleftrightarrow{FT} \frac{1}{|a|} X\left(\frac{\omega}{a}\right).}$